

Neutral-meson oscillations with torsion

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We propose a simple mechanism that may explain the observed particle-antiparticle asymmetry in the Universe. In the Einstein-Cartan-Sciama-Kibble theory of gravity, the intrinsic spin of matter generates spacetime torsion. Classical Dirac fields in the presence of torsion obey the nonlinear Hehl-Datta equation which is asymmetric under a charge-conjugation transformation. Accordingly, at extremely high densities that existed in the very early Universe, fermions have higher effective masses than antifermions. As a result, a meson composed of a light quark and a heavy antiquark has a lower effective mass than its antiparticle. Neutral-meson oscillations in thermal equilibrium therefore favor the production of light quarks and heavy antiquarks, which may be related to baryogenesis.

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The Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity naturally extends general relativity (GR) to include the intrinsic angular momentum of matter [1, 2]. In this theory, the spin of Dirac fields is a source of the torsion tensor S^k_{ij} , which is the antisymmetric part of the affine connection: $S^k_{ij} = \Gamma^k_{[ij]}$. The Lagrangian density for a free Dirac spinor ψ with mass m is given by $\mathcal{L}_D = \frac{i\sqrt{-g}}{2}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi$, where g is the determinant of the metric tensor g_{ik} and the semicolon denotes a covariant derivative with respect to the affine connection Γ^k_{ij} . We use units in which $\hbar = c = k_B = 1$. Varying \mathcal{L}_D with respect to the spinor adjoint conjugate $\bar{\psi}$ gives the Dirac equation $i\gamma^k(\psi_{;k} + \frac{1}{4}C_{ijk}\gamma^i\gamma^j\psi) = m\psi$, where the colon denotes a Riemannian covariant derivative (with respect to the Christoffel symbols) and $C_{ijk} = S_{ijk} + S_{jki} + S_{kji}$ is the contortion tensor [2, 3]. Varying the total (gravity plus matter) Lagrangian density $-\frac{R\sqrt{-g}}{2\kappa} + \mathcal{L}_D$ with respect to C_{ijk} gives the Cartan relation [1] between the torsion tensor and the Dirac spin tensor $s^{ijk} = \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_D}{\delta C_{ijk}} = \frac{1}{2}e^{ijkl}j_{Al}$, where the tensor $e^{ijkl} = \frac{\epsilon^{ijkl}}{\sqrt{-g}}$, ϵ^{ijkl} is the Levi-Civita permutation symbol and $j^k_A = \bar{\psi}\gamma^5\gamma^k\psi$ is the axial fermion current [2, 3]. Substituting this quadratic (in spinor fields) relation to the Dirac equation gives the cubic Hehl-Datta equation for ψ [2, 3]:

$$i\gamma^k\psi_{;k} = m\psi - \frac{3\kappa}{8}j_{Ak}\gamma^5\gamma^k\psi. \quad (1)$$

For a spinor with electric charge e in the presence of the electromagnetic potential A_k , we must replace $\psi_{;k}$ by $\psi_{;k} - ieA_k\psi$:

$$i\gamma^k\psi_{;k} + eA_k\gamma^k\psi = m\psi - \frac{3\kappa}{8}(\bar{\psi}\gamma^5\gamma_k\psi)\gamma^5\gamma^k\psi. \quad (2)$$

The charge conjugate ψ^c of a spinor ψ is defined as $\psi^c = -i\gamma^2\psi^*$ [4]. The complex conjugate of (2) leads to [5]

$$i\gamma^k\psi^c_{;k} - eA_k\gamma^k\psi^c = m\psi^c + \frac{3\kappa}{8}(\bar{\psi}^c\gamma^5\gamma_k\psi^c)\gamma^5\gamma^k\psi^c. \quad (3)$$

Comparing (2) with (3) shows that ψ and ψ^c correspond to the opposite values of e , as expected from a charge-conjugation transformation. They also satisfy different (classical) field equations because of the opposite signs of the corresponding Hehl-Datta cubic terms relative to the mass term. Therefore, the classical Hehl-Datta equation leads to a charge-conjugation asymmetry between fermions and antifermions [5]. This equation solved for fermion plane waves in the approximation of Riemann flatness gives the energy levels for a free fermion, $\omega = m + \epsilon$, where

$$\epsilon = \alpha\kappa N, \quad (4)$$

N is the inverse normalization of the spinor's wave function, and $\alpha \sim 1$ is a constant [5, 6]. These levels are higher than for the corresponding antifermion, $\omega = m - \epsilon$ [5]. Since fermions have higher energy levels than antifermions due

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to the charge-conjugation asymmetric Hehl-Datta term, they are effectively more massive (in dispersion relations) and decay faster. Such a difference between fermions and antifermions plays is significant only at extremely high densities where the contributions to the energy-momentum tensor from the spin density and from the energy density are on the same order [7, 8]. In almost all physical situations, the ECSK gravity reduces to GR in which the Hehl-Datta term vanishes¹ and the field equations are charge-conjugation symmetric.²

We now investigate how the above asymmetry of the Hehl-Datta equation affects the masses of mesons. The effective masses due to torsion of a quark, m_q , and an antiquark, $m_{\bar{q}}$, are given by

$$m_q = m_q^{(0)} + \epsilon, \quad (5)$$

$$m_{\bar{q}} = m_{\bar{q}}^{(0)} - \epsilon, \quad (6)$$

where $m_q^{(0)}$ and $m_{\bar{q}}^{(0)}$ are the corresponding masses in GR, where the Hehl-Datta term is absent. The sum of these masses is not affected by torsion, $m_q + m_{\bar{q}} = m_q^{(0)} + m_{\bar{q}}^{(0)}$. The simplest toy relation for the effective mass of a ground-state meson, $m_{q\bar{q}}$, is

$$m_{q\bar{q}} = m_q + m_{\bar{q}} - \frac{m_q m_{\bar{q}}}{\mu}, \quad (7)$$

where the last term on the right-hand side is an interaction term with a negative sign due to the fact that a meson is a bound state of a quark and an antiquark. The strength of an interaction between a quark and an antiquark is given by a mass-dimension parameter μ . Without torsion, the corresponding mass of a meson would be

$$m_{q\bar{q}}^{(0)} = m_q^{(0)} + m_{\bar{q}}^{(0)} - \frac{m_q^{(0)} m_{\bar{q}}^{(0)}}{\mu}. \quad (8)$$

The relations (5), (6), (7) and (8) give

$$m_{q\bar{q}} = m_{q\bar{q}}^{(0)} + \frac{\epsilon}{\mu}(m_q^{(0)} - m_{\bar{q}}^{(0)}). \quad (9)$$

If a meson is composed of a quark and its Dirac-adjoint antiquark, then $m_q^{(0)} = m_{\bar{q}}^{(0)}$. In this case, the Hehl-Datta term does not affect the mass of such a meson, $m_{q\bar{q}} = m_{q\bar{q}}^{(0)}$. As a result, the mass of the corresponding antimeson is the same as the mass of the meson. If, however, a meson is composed of a quark and an antiquark with different flavors, then the mass of the corresponding antimeson is not equal to the mass of the meson. For example, for neutral B mesons,

$$m_{B_0} = m_{d\bar{b}} = m_{d\bar{b}}^{(0)} + \frac{\epsilon}{\mu}(m_d^{(0)} - m_{\bar{b}}^{(0)}) = M_{B_0} - \Delta_{bd}, \quad (10)$$

$$m_{\bar{B}_0} = m_{b\bar{d}} = m_{b\bar{d}}^{(0)} + \frac{\epsilon}{\mu}(m_b^{(0)} - m_{\bar{d}}^{(0)}) = M_{B_0} + \Delta_{bd}, \quad (11)$$

where

$$M_{B_0} = m_{b\bar{d}}^{(0)} \quad (12)$$

and

$$\Delta_{bd} = \frac{\epsilon}{\mu}(m_b^{(0)} - m_{\bar{d}}^{(0)}) > 0. \quad (13)$$

The relations (10) and (11) use $m_b^{(0)} = m_{\bar{b}}^{(0)}$, $m_d^{(0)} = m_{\bar{d}}^{(0)}$ and $m_{b\bar{d}}^{(0)} = m_{d\bar{b}}^{(0)}$.

¹ The Hehl-Datta equation corresponds to the axial-axial four-fermion interaction term in the Dirac Lagrangian. If spinor fields, such as quarks, have a nonzero vacuum expectation value (form condensates), then this term can be the source of the observed small, positive cosmological constant [9].

² Torsion may also introduce an effective ultraviolet cutoff in quantum field theory for fermions [8]. The solutions of the nonlinear equations for the two-point function of a nonlinear spinor theory of [10] exhibit self-regulation of its short-distance behavior [11]. The propagator of a similar theory based on the Hehl-Datta equation should thus also be self-regulated.

Neutral-meson oscillations transform neutral particles with nonzero internal quantum numbers into their antiparticles. For example, B_0 mesons transform into their antiparticles \bar{B}_0 through the weak interaction. In thermal equilibrium at temperature T , the particle number densities n of these mesons are related by

$$\frac{n_{\bar{B}_0}}{n_{B_0}} = e^{-(m_{B_0} - m_{\bar{B}_0})/T} = e^{-2\Delta_{bd}/T}. \quad (14)$$

If $\Delta_{bd} \ll T$ then

$$\frac{n_{B_0} - n_{\bar{B}_0}}{n_{B_0} + n_{\bar{B}_0}} \approx \frac{\Delta_{bd}}{T}. \quad (15)$$

$B_0 - \bar{B}_0$ oscillations in thermal equilibrium therefore produce more d quarks than \bar{d} antiquarks and more \bar{b} antiquarks than b quarks. Generally, neutral-meson oscillations in thermal equilibrium favor the production of light quarks and heavy antiquarks. Such an asymmetry caused by the fermion-torsion coupling may be related to baryogenesis [5].

Since the inverse normalization N of a Dirac spinor is on the order of the cube of its energy scale and such a scale in the early Universe is given by the temperature of the Universe, we have $N \sim T^3$ [5]. Substituting this relation to (4), (13) and (15) gives

$$\alpha = \frac{n_{B_0} - n_{\bar{B}_0}}{n_{B_0} + n_{\bar{B}_0}} \approx \frac{\kappa(m_b^{(0)} - m_d^{(0)})T^2}{\mu}, \quad (16)$$

which then gives

$$T \approx m_{\text{Pl}} \left(\frac{\alpha\mu}{m_b^{(0)} - m_d^{(0)}} \right)^{1/2} \approx m_{\text{Pl}} \left(\frac{\alpha\mu}{m_b^{(0)}} \right)^{1/2}. \quad (17)$$

Putting $m_b^{(0)} \approx 4 \text{ GeV}$, $\alpha \approx 10^{-10}$, which is on the order of the observed baryon-to-entropy ratio, and $\mu \approx 200 \text{ MeV}$, which is near the QCD scale parameter of the SU(3) gauge coupling constant, gives $T \approx 10^{-6} m_{\text{Pl}} \approx 10^{12} \text{ GeV}$ (m_{Pl} is the reduced Planck mass). This value is near the freeze-out temperature found in [5].

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